

DESIGN OPTIMIZATION OF POLYNOMIAL FLEXURAL HINGES FOR ENHANCED ULTRASONIC VIBRATION-ASSISTED MACHINING

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Abstract. The trend in vibrator module design for vibration-assisted machining involves utilizing flexural hinges as vibrational transmitters from the actuator to the cutting tool. Polynomial flexural hinges represent a type that provides greater flexibility to designers, as polynomial hinges can encompass properties of other hinge types. This study aims to demonstrate the influence of polynomial hinge design parameters through finite element analysis. These simulations were used to analyze the effects of polynomial hinge design parameters i.e., polynomial orders, hinge thickness, and hinge length on deflection at the tooltip and hinge stress. A full factorial design with five levels for each parameter was implemented. Increasing polynomial orders and longer hinges increased deflection by twofold from 0.9 to 2.04 μm , and hinge stress by similar order of magnitude, ranging from 61 to 112 MPa. Within the combinations studied, grey relation analysis indicated the optimal conditions for a combination polynomial hinge designs with a polynomial order n of 2, a thickness of 4 mm, and a length of 6 mm. This combination resulted in a tool deflection of 1.47 μm and stress of 39.1 MPa. Consequently, this study is expected to contribute to the knowledge about polynomial hinge design for the vibration-assisted machining, enabling its application across various industries.

Keywords: Ultrasonic vibration-assisted machining, Flexure hinge, Polynomial hinge, Deflection, Finite element analysis

1. Introduction

Despite the proven cutting quality of Ultrasonic Vibration-Assisted Machining (UVAM) [1], improving the cutting tool's vibration settings of this advanced machining technique remains a major challenge. A critical aspect is the mechanism for transmitting actuated vibrations to the cutting tool to ensure controllable tool oscillation. Achieving precise control over these vibrations is essential for optimizing cutting performance. Additionally, ensuring that the transmitted vibrations are consistent and stable can significantly impact the quality of the machined surface. The control of oscillations at the tool tip can be achieved if the vibrations are transmitted consistently and stably. Therefore, a reliable connection between the piezoelectric and the tool tip is essential to ensure better vibration transmission.

UVAM has shown a trend of incorporating flexure hinges into the vibration tool to improve the vibration transmission [2–4]. A flexure hinge is a specialized mechanical component that enables limited angular motion through the elastic deformation of the material. Integrating a flexure hinge into the UVAM vibration module facilitates vibration transmission from the piezoelectric device to the tool tip [5–7]. The piezoelectric device generates high-frequency, small-amplitude vibrations, serving as the vibration source in UVAM [8]. These vibrations cause the tool tip to move in small reciprocating motions (1D VAM) or elliptical motions (2D VAM), with the centroid moving in the cutting direction [9] Figure 1. This oscillatory motion during cutting leads to improved cutting outcomes [10–13]. However, the effective vibration for this scenario occurs when the amplitude exceeds $1\ \mu\text{m}$ [14]. Without a flexure hinge, the vibration tool driven by piezoelectric vibrations has an amplitude of only about $0.001\ \mu\text{m}$ [15]. Thus, implementing a flexure hinge in the UVAM module is essential to enhance the amplitude of the tool's vibration.

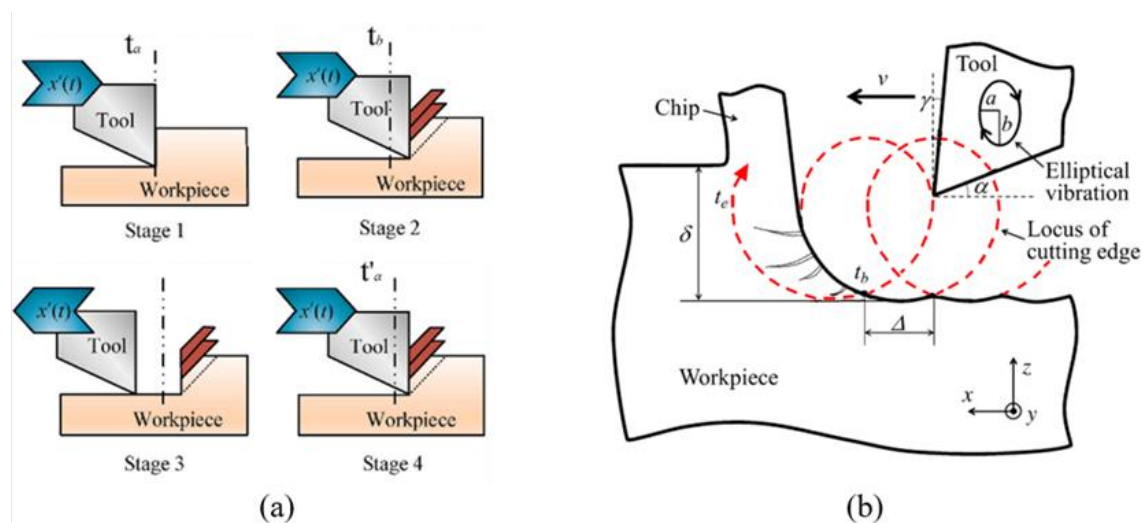


Figure 1 UVAM Mechanisms: (a) 1D UVAM Movement and (b) 2D UVAM Technique. The control of tool vibrations significantly impacts cutting performance and outcomes.

Due to the broad and varied applications, the design of flexure hinges varies according to the specific needs of the process. Commonly used types include semi-circular hinges, elliptical hinges, corner-filletted hinges, and polynomial hinges [16–18] Figure 2. They have unique characteristics that make them suitable for particular uses. Semi-circular hinges are designed with a curved profile that provides uniform stress distribution. Elliptical hinges feature an elongated, oval shape that enhances flexibility and range of motion. Corner-filletted hinges incorporate rounded corners into their design, which helps to reduce stress concentrations that can occur at sharp edges.

Polynomial hinges stand out due to their design flexibility [19]. Their shape can be adjusted based on polynomial functions, allowing engineers to fine-tune the hinge's performance characteristics

according to specific needs. This adaptability makes polynomial hinges particularly versatile, as they can be customized for various applications, from simple to complex motions [20].

This investigation focuses on the polynomial hinge, which features a relatively simple shape, is easy to manufacture, and is commonly used in practice [21]. In contrast to alternative flexure hinges, the polynomial hinge is designed to align with its specified polynomial order, thereby offering enhanced design flexibility and enabling shape adjustments to be made in accordance with specific requirements. The function used to determine the contour of the polynomial hinge is [18].

$$h_n(x) = h + \frac{(H - h)}{\left(\frac{1}{2}\right)^n} x^n$$

where h is hinge thickness, H is thickness and n is the polynomial order.

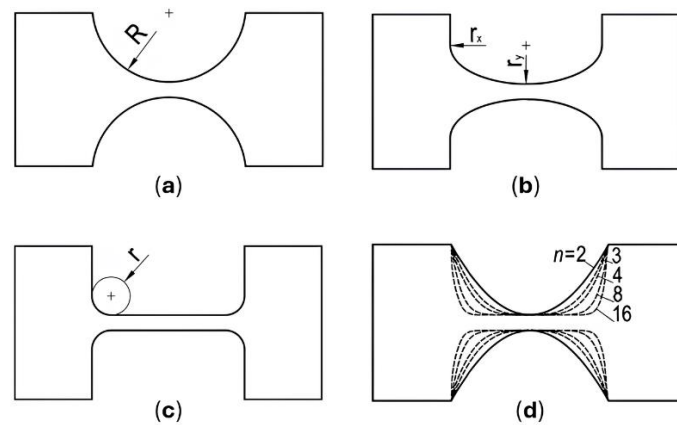


Figure 2 The contours of hinges in several types include (a) semi-circular, (b) elliptical hinges, (c) corner-filletted hinges, and (d) polynomial hinges with orders of $n = 2, 3, 4, 8,$ and 16 .

The influence of individual parameter of polynomial notch hinge has been demonstrated in previous studies [22], but the combined effects of multiple design parameters on the deformation and stress of a polynomial hinge and its impact on the deformation level in ultrasonic vibration-assisted machining (UVAM) is remained unclear. Therefore, this study aims to propose an optimal polynomial hinge design by combining these parameters to achieve significant deformation with manageable stress levels. The analysis provides insights into the hinge's ability to maintain structural integrity while facilitating the desired oscillatory motion. The findings are expected to contribute to the development of more efficient and reliable UVAM systems. Ultimately, the results will offer valuable guidelines for designing polynomial hinges that meet the specific demands of advanced manufacturing processes.

2. Method

The deformation and stress characteristics of the polynomial notch hinge were investigated using data obtained from finite element analysis (FEA). The analysis was conducted in Ansys Workbench 2021 on simulated models generated through the full factorial method, incorporating three design parameters (polynomial order, hinge thickness and hinge length) with five levels each Table 1. The simulated models were designed in accordance with the proven non-resonant UVAM module proposed by Hakim, Lukman Nul [23].

Table 1 Design parameters and their levels for the simulated polynomial hinge model

Parameter	Level 1	Level 2	Level 3	Level 4	Level 5
Orders n	$2n$	$3n$	$4n$	$8n$	$16n$
Hinge thickness	2 mm	2.5 mm	3 mm	3.5 mm	4 mm
Hinge length	5.5 mm	6 mm	6.5 mm	7 mm	7.5 mm

Prior to the performance simulation, each model was subjected to modal analysis with the objective of determining its natural frequencies. It was determined that the 20 kHz frequency was relatively distant from the identified natural frequencies of the simulated models. Accordingly, this frequency was selected for the ultrasonic vibration to investigate hinge deformation and stress using explicit dynamic simulation analysis. The external loads ($F_x = 4.466$ N, $F_y = 139.024$ N, and $F_z = 80.282$ N) were applied at the tip of the model, simulating the cutting force during UVAM process. Furthermore, presents the other important simulation settings selected for both modal and explicit analysis.

The FEA was conducted to analyze the response of a flexure hinge to vibrational forces. The material selected for the simulation was AISI 1040, known for its mechanical properties suitable for high-stress applications. A refined tetrahedral mesh with a size of 0.5 mm was applied specifically around the hinge area to capture the intricate deformation characteristics, while a coarser mesh size of 5 mm was used for other regions to balance computational efficiency with accuracy. The simulation was conducted in five steps of vibration, with a frequency set at 20,000 Hz. The PB10.18 piezoelectric was employed as the vibration source. To replicate real-world machining conditions, a force of 3500 N was applied during the simulation.

3. Results and Discussion

The impacts of varying polynomial order, hinge thickness, and hinge length are illustrated in Figure 2. A high-order polynomial, along with a thin and long hinge, results in larger hinge deformation. Specifically, increasing the polynomial order from 1 to 5 was found to double the hinge deformation, demonstrating the significant influence of the polynomial order on the flexure performance. A similar trend was observed with longer hinges, which allow for more significant deformation due to the longer thin section of the hinge, thereby enhancing the overall flexibility of the hinge.

Conversely, increasing the hinge thickness from 2 to 4 mm significantly reduced the hinge deformation from 2.7 μm to 0.5 μm . This reduction highlights the inverse relationship between hinge thickness and deformation, where a thicker hinge provides more resistance to bending, thus limiting the extent of deformation. These findings underscore the critical balance between hinge thickness and length, and polynomial order in designing an effective hinge for UVAM applications. By optimizing these parameters, the performance of the UVAM system can be significantly enhanced, leading to more precise and efficient machining processes.

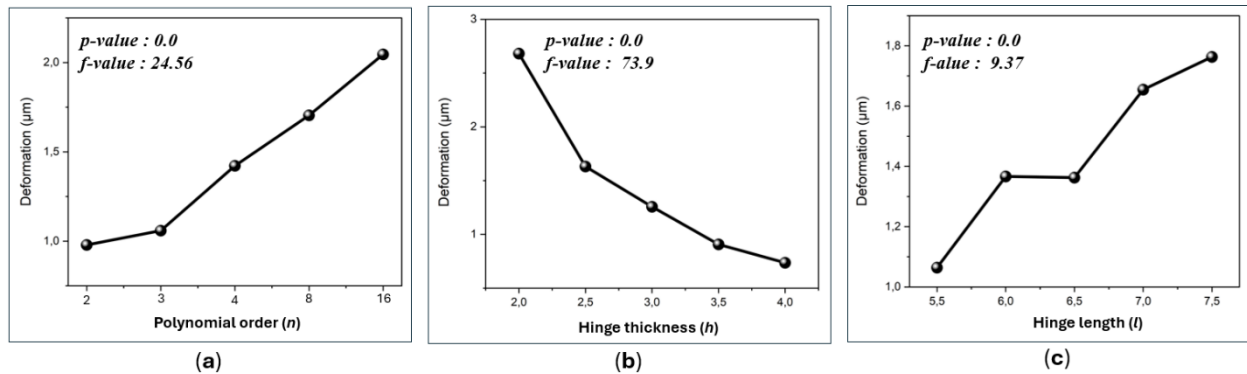


Figure 3 Correlation of (a) polynomial order, (b) hinge thickness, and (c) hinge length to the deformation of polynomial hinges simulated by FEA. Large deformation is achieved with higher polynomial order, thinner hinges, and longer hinges.

The FEA simulation was extended to analyse the stress occurred at the vicinity of the hinge. Figure 3 displays the global trends of changing polynomial order, thickness and length of the polynomial notch hinge design. Positive correlation appears when high order polynomial hinge and longer hinge was used. Nearly linear relationship between polynomial order and hinge stress was observed when the polynomial order increased from 1 to 5 with approximate 90 % hinge stress increment. Meanwhile, lesser hinge stress increment (90%) identified when additional 2 mm length was added from 5.5 mm. Inversely, increasing hinge thickness exhibits negative trend to the hinge stress. The hinge stress was pressed down from approx. 150 MPa to nearly 50 MPa when the hinge thickness change from 2 to 4 mm.

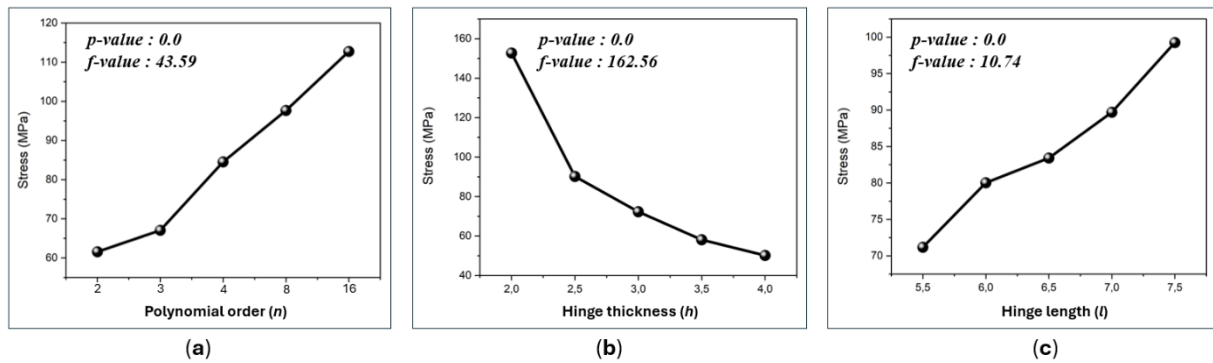


Figure 4 The impact of polynomial order, hinge thickness, and hinge length on the concentrated stress around the hinge. Lower polynomial orders, thicker hinges, and shorter hinges are preferred to minimize hinge stress.

These results highlight the intricate balance required in the design of polynomial notch hinges for UVAM applications. The increase in hinge stress with higher polynomial orders and longer hinges can be attributed to the increased deformation, which induces greater stress in the material. On the other hand, thicker hinges provide more material to distribute the stress, thereby reducing the overall stress levels. This information is crucial for optimizing the design parameters to achieve the desired performance characteristics. By carefully adjusting the polynomial order, hinge thickness, and hinge length, it is possible to tailor the hinge design to specific application requirements, ensuring both high deformation and manageable stress levels. This optimization is essential for enhancing the durability and efficiency of UVAM systems, ultimately leading to more precise and reliable machining processes.

The grey relational analysis results on the performance of the simulated models are presented in Table 2. The top-ranked model, featuring an optimal combination of large deformation and minimal hinge stress, is the polynomial hinge with a polynomial order of 2, a hinge thickness of 4 mm, and a hinge length of 6 mm. This model produced a hinge deformation of 1.47 μm and a hinge stress of 39.1 MPa.

Table 2 The ranking of polynomial hinge models based on grey relational analysis results

Rank	n	h	l	$D x_i(k)$	$S x_i(k)$	$D \Delta_i$	$S \Delta_i$	$D \gamma_i(k)$	$S \gamma_i(k)$	γ_i
1	2	4	6	0.053	0.996	0.947	0.004	0.346	0.992	0.669
2	3	2	6.5	0.000	1.000	1.000	0.000	0.333	1.000	0.667
3	16	2	7.5	1.000	0.000	0.000	1.000	1.000	0.333	0.667
4	2	4	5.5	0.092	0.988	0.908	0.012	0.355	0.977	0.666
5	3	4	5.5	0.059	0.990	0.941	0.010	0.347	0.981	0.664
6	2	4	7	0.056	0.990	0.944	0.010	0.346	0.981	0.664
7	2	4	7.5	0.066	0.988	0.934	0.012	0.349	0.977	0.663
8	16	2.5	5.5	0.062	0.987	0.938	0.013	0.348	0.975	0.662
9	2	4	6.5	0.042	0.986	0.958	0.014	0.343	0.972	0.658
10	4	4	5.5	0.073	0.980	0.927	0.020	0.350	0.962	0.656

The significant hinge deformation of 1.47 μm suggests that the vibration amplitude of the tool is well within the desired range, which is critical for achieving the benefits of UVAM, such as reduced cutting forces and improved surface finish. The low hinge stress of 39.1 MPa ensures that the hinge can withstand the operational stresses without risk of failure, thus enhancing the longevity and stability of the UVAM system. Compared to the notch hinge with a radius of 3 mm, a hinge thickness of 2 mm, and an overall thickness of 16 mm, which results in a deformation of 1.94 μm and a stress of 64.2 MPa, the polynomial hinge with a polynomial order of n 2, a hinge thickness of 2 mm, and a length of 5.5 mm exhibits a higher deformation of 3.09 μm . However, this 37% increase in deformation is also accompanied by a 25% increase in stress compared to the notched hinge. Given these performance characteristics, this polynomial hinge design is highly promising for future development of UVAM vibration modules. It offers a robust solution that can potentially improve the efficiency and effectiveness of UVAM applications, particularly in precision machining of hard-to-cut materials.

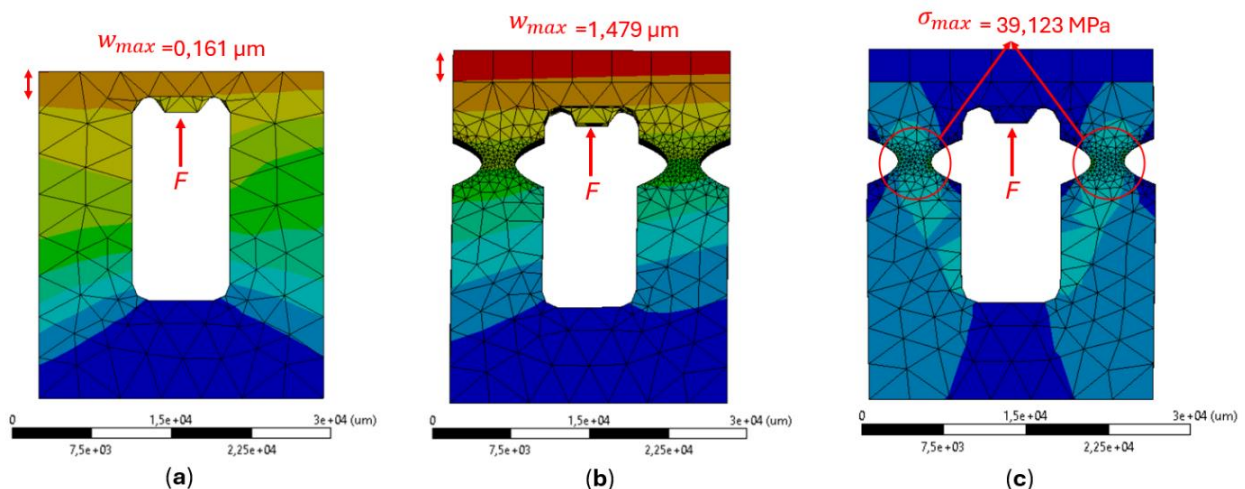


Figure 5 Performance comparison of UVAM modules: (a) Without a flexure hinge, showing limited deformation; (b) With the optimized polynomial hinge, demonstrating enhanced deformation capabilities; (c) Maximum hinge stress for the optimized model, illustrating the stress distribution.

4. Conclusions

This study has demonstrated the significant impact of asymmetrical flexure hinge design to the improved oscillation UVAM vibration module. By integrating polynomial notch hinges, the transmission of high-frequency vibrations from the piezoelectric device to the cutting tool can be effectively enhanced. The finite element analysis results highlighted that the polynomial order, hinge thickness, and hinge length are critical parameters influencing hinge deformation and stress. Higher polynomial orders and longer hinges resulted in greater deformation, while thicker hinges reduced stress concentration. The optimal combination of design parameters, as determined by grey relational analysis, consisted of a polynomial order of 2, a hinge thickness of 4 mm, and a hinge length of 6 mm. This combination yielded a hinge deformation of 1.47 μm and a hinge stress of 39.1 MPa. This balance ensures that the vibration amplitude is within the desired range for effective UVAM performance, while the stress levels remain manageable, thus enhancing the durability of the hinge. Overall, these findings provide valuable guidelines for designing efficient and reliable UVAM modules, paving the way for advancements in precision machining of challenging materials.

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